

IS MASS QUANTIZED?

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Abstract

The cosmological constant combined with Planck's constant and the speed of light implies a quantum of mass of approximately 2×10^{-65} g. This follows either from a generic dimensional analysis, or from a specific analysis where the cosmological constant appears in 4D spacetime as the result of a dimensional reduction from higher dimensional relativity (such as 5D induced-matter and membrane theory). In the latter type of theory, all the particles in the universe can be in higher-dimensional contact.

1 Introduction

Observations of the cosmic microwave background, the dynamics of galaxies and the gravitational lensing of remote astronomical sources such as quasars imply that 99% of the material in the universe is dark matter.¹ Of this, a significant fraction appears to be due to the equivalent density of the “vacuum”. This in general relativity is measured by $\Lambda c^2/8\pi G$ where Λ is the cosmological constant, c is the speed of light and G is the gravitational constant.² The size of the first parameter is $\Lambda \simeq 3 \times 10^{-56} \text{ cm}^{-2}$ approximately.^{3,4} (Equivalently, the observed universe has an intrinsic length scale of order

10^{28} cm, or an age of order 10^{10} years, with a vacuum density of order 10^{-29} g cm $^{-3}$.) It is now widely believed that the cosmological constant of general relativity is a parameter which is derived from the reduction to 4D of higher-dimensional theories whose motivation is to unify gravity with the interactions of particle physics.² These include 10D supersymmetry, 11D supergravity and 26D string theory. They should provide a natural place for the quantum of action as measured by Planck's constant h . The basic extension of 4D Einstein theory and the low-energy limit of higher-D theories is the modern incarnation of (non-compact) 5D Kaluza-Klein theory. This has been intensively studied recently, under the names induced-matter theory^{5,6} and membrane theory.^{7,8} Both admit a fifth force which may be relevant to the interactions of particles^{9,10} and both allow us to view massive particles in spacetime as massless or photon-like in the larger manifold^{11,12} (i.e., timelike 4D paths may be viewed as 5D null geodesics). Also, the field equations of both versions of 5D relativity have recently been shown to be equivalent¹³. In regard to comparison of the 5D field equations with observations, Campbell's theorem guarantees an embedding of the 4D Einstein equations and their Newtonian limit¹⁴, and an explicit calculation shows that the classical tests of relativity are satisfied¹⁵. In comparison with cosmologi-

cal data, Campbell's theorem means that the standard models are recovered, and there are explicit 5D solutions which represent the present universe^{2,16} as well as the early universe with inflation^{2,17} and an effective cosmological constant.

These results mean that we are now in a position to take a fresh look at the constants c , G , h and Λ . In what follows, we will do this first in a generic sense, using only dimensional analysis; and then we will collect some technical results from 5D relativity to give a more specific account. Both approaches will be seen to imply that mass is quantized at the level of approximately 2×10^{-65} g.

2 Cosmological Constant and Mass Scales

In this section, we will use the fundamental parameters and then a more detailed analysis to show that the cosmological constant implies two distinct mass scales. Of these, one is a minimum and so defines a quantum of mass. Some of what follows may be familiar, but in addressing such a fundamental issue we wish to ensure that the traditional approach and the new one are compatible.

Dimensional analysis is an elementary group-theoretic technique^{18–22}. It is related to the fact that the equations of physics are homogeneous in their physical dimensions, which implies that they can be written in terms of dimensionless quantities if we so wish (this is the basis of modelling theory). The technique involves the ability to transform quantities of different physical types to ones of the same physical dimensions using the constants (this is the basis of using $x^0 \equiv ct$ to transform the time to a length coordinate in relativity). The technique also implies the freedom to choose units, which in mechanics means fiducial values for the base physical dimensions M , L , T of mass, length and time (this is the basis of the convenient choice $c = 1$, $G = 1$ in relativity). However, dimensional analysis is a generic technique, without detailed knowledge of the underlying theory to which it is applied (which is why it fails to determine dimensionless factors such as integers and π). Also, it is problematic in application when the constants have physical dimensions which “overlap” or are degenerate²¹. This is the case presently being encountered in cosmology with the recognition of Λ as a fundamental constant. To illustrate what is involved here, we simply have to realize that from the 4 constants c , G , h and Λ we can form two different masses

$$m_P \equiv (h/c) (\Lambda/3)^{1/2} \simeq 2 \times 10^{-65} \text{ g} \quad (1)$$

$$m_E \equiv (c^2/G) (3/\Lambda)^{1/2} \simeq 1 \times 10^{56} g \quad . \quad (2)$$

Here the two masses are relevant to quantum and gravitational situations, and so may be designated by the names Planck and Einstein respectively. [To avoid confusion, it can be mentioned that the mass $m_{PE} \equiv (hc/G)^{1/2} \simeq 5 \times 10^{-5}$ g which is sometimes called the Planck mass does not involve Λ and mixes h and G . From the viewpoint of higher-dimensional field theory as outlined below, this is equivalent to mixing gauges and is ill-defined, possibly explaining why this mass is not manifested in nature²⁰.] The mass (2) is straightforward to interpret: it is the mass of the observable part of the universe, equivalent to 10^{80} baryons of 10^{-24} g each. The mass (1) is more difficult to interpret: it is the mass of a quantum perturbation in a spacetime with very small local curvature, measured by the astrophysical value of Λ as opposed to the one sometimes inferred from the zero-point or vacuum fields of particle interactions. We are fully cognizant of this mismatch, which is commonly called the cosmological-constant problem^{1,22–26}. [Its essence is that if one believes m_{PE} to be a physical mass scale then Λ in (1) has to be larger than that in (2) by 10^{120} or so.] However, in our approach the cosmological-constant problem becomes moot, because even if Λ were larger

in localized regions of space² or in the early universe⁴, its astrophysical value is still a minimum and so the mass (1) is still the smallest one possible.

Higher-dimensional field theory provides not only a rationale for the Λ we measure in spacetime but also an account of 4D dynamics (based on solutions of the field equations and the equations of motion). In the basic 5D theory, the “separation” (squared) between two nearby points is given by the line element $dS^2 = g_{AB}dx^A dx^B$, which contains that of general relativity $ds^2 = g_{\alpha\beta}dx^\alpha dx^\beta$. (Here $A, B = 0, 1, 2, 3, 4$ and $\alpha, \beta = 0, 1, 2, 3$ for coordinates $x^0 = ct$, $x^{123} = xyz$ and $x^4 = l$. There is a summation over indices repeated in the metric tensor and the coordinate elements.) For the induced-matter version of the theory most work has been done using the canonical form of the metric⁶, while for the membrane version most has been done using the warp form of the metric⁷. As noted above, the two versions of 5D relativity are mathematically equivalent at a general level. However, the results may not be physically equivalent, because both approaches depend on a choice of coordinates or gauge. [This is because the 5D group of transformations $x^A \rightarrow \bar{x}^A(x^B)$ is wider than the 4D group $x^\alpha \rightarrow \bar{x}^\alpha(x^\beta)$, so spacetime physics can change under an l -dependent change of gauge: see refs. 2, 11, 20.] With respect to this and a wish to understand the masses (1), (2) noted above,

it is instructive to introduce a new gauge. We call this the Planck gauge. And for reasons which will become apparent, we rename the canonical frame the Einstein gauge. The two are specified respectively by

$$dS^2 = (L/l_P)^2 ds^2 - (L/l_P)^4 dl_P^2 \quad (3)$$

$$dS^2 = (l_E/L)^2 ds^2 - dl_E^2 \quad . \quad (4)$$

Here L is a constant length introduced to give consistency of physical dimensions. However, it turns out to have great relevance to our present discussion, because a reduction of the field equations in 5D to their counterparts in 4D (which include Einstein's equations) shows that L is related to the cosmological constant via $\Lambda = 3/L^2$ (see refs. 2, 6, 17, 23). Thus cosmological data imply $L \simeq 1 \times 10^{28}$ cm. The extra coordinates l_P , l_E in (3), (4) may be shown by another reduction of the equations of motion to be related to the Compton wavelength and Schwarzschild radius of a test particle of mass m via $l_P = h/mc$ and $l_E = Gm/c^2$ (see refs. 2, 20, 27; this can be appreciated directly by noting that with these identifications, the first parts of the elements of the 5D actions specified by dS involve the elements of the 4D action $mcds$ specified by the proper time ds). The two gauges just stated are

related by the simple coordinate transformation $l_E = L^2/l_P$, which can be used to go between them. They are not arbitrary, of course, but chosen with care. They lead to dramatic simplifications in the underlying field equations and equations of motion, and represent the most convenient way to embed the physics of 4D spacetime in a 5D manifold.

To illustrate the physics inherent in (3) and (4), let us recall that particles travelling on paths in 4D with $ds^2 \geq 0$ can be regarded as travelling on null geodesics in 5D with $dS^2 = 0$ ^{11,12}. The last statement means that, in some sense, particles are in causal contact in 5D. (They are analogous to photons in 4D, which can be viewed as connecting events which are separated in ordinary 3D space.) This condition with (3) means that the latter can be rewritten and integrated to yield

$$\int d(L/l_P) = (1/h) \int mcds \quad . \quad (5)$$

Here we know that the conventional action is quantized and equal to nh where n is an integer. Thus $L/l_P = n$. This says that the Compton wavelength of the particle cannot take on any value, but is restricted by the typical dimension of the (in general curved) spacetime in which it exists. Putting back the relevant parameters, the last relation says that $m = (nh/c)(\Lambda/3)^{1/2}$. For the groundstate with $n = 1$, there is a mini-

mum mass $m_P = (h/c) (\Lambda/3)^{1/2} \simeq 2 \times 10^{-65}$ g. This is the same as (1) above.

A similar procedure to that of the preceding paragraph can be followed for the Einstein gauge (4). However, a notable difference occurs in that the relation analogous to (5) is now

$$\int (L/l_E) dl_E = \int ds \quad . \quad (6)$$

Here we do not have any evidence that the line element by itself is quantized, so the discreteness which is natural for the Planck gauge does not carry over to the Einstein gauge. However, in the Planck gauge the condition $L/l = n$ could have been used to reverse the argument and deduce the quantization of the action from the quantization of the fifth dimension, implying that the latter may be the fundamental assumption. Let us take this in the form $L/l_E = n$. [This by (6) then implies $dl/ds = 1/n$, which also by (5) holds in the Planck gauge. The velocity in the fifth dimension is related to electric charge in certain approaches to 5D relativity, including the early one of Klein².] Then putting back the relevant parameters, we obtain $m_E = (c^2/nG) (3/\Lambda)^{1/2}$. For the groundstate with $n = 1$, there is a maximum mass $m_E = (c^2/G) (3/\Lambda)^{1/2} \simeq 1 \times 10^{56}$ g. This is the same as (2) above.

In summary, astrophysical data indicate that we should add the cosmological constant Λ to the suite of fundamental physical parameters, which implies a mass (1) related to Planck's constant of approximately 2×10^{-65} g and a mass (2) related to the gravitational constant of approximately 1×10^{56} g. Both can be understood at a deeper level if the world has more than the 4 dimensions of spacetime. In the prototypical 5D theory, Λ is related to a length which scales the $(4 + 1)$ parts of the manifold. The latter can most conveniently be described by the Planck gauge (3) and Einstein gauge (4). These for null 5D paths lead to relations (5) and (6), which imply that mass is quantized.

3 Discussion

Our main result, that there is a minimum mass of approximately 2×10^{-65} g, raises many questions of both a theoretical and practical nature. For example, if particles of this mass are involved in interactions, the range of the latter would be large but finite. The noted mass is tiny, even by the standards of particle physics. This explains why mass is apparently unquantized at the levels we have been able to examine, but also means that a direct test

involving current accelerators is impractical. However, the existence of this quantum rests on the assumption that paths in 5D are null, and this may provide an indirect test of the approach. It is already known that the photons of the cosmic microwave background have the same temperature to an accuracy of 1 part in 10^5 , even though according to standard models the parts of the universe where they originated were out of (4D) causal contact at early times. The conventional way to explain this is, of course, via inflation (an early period of rapid expansion). But it is not clear if this also explains the uniformity of the properties of massive particles as revealed by the spectroscopy of remote astronomical sources such as quasars. An alternative view, which needs analysis, is that the universe in 4D appears uniform because all of its constituents are in causal contact in 5 (or more) dimensions. A related route to testing the approach outlined above involves work in the laboratory. The classical double-slit experiment, and others like it which show quantum interference, should be revisited, to see if the apparently baffling behaviour of electrons in ordinary space is due to the fact that they are in causal contact in higher dimensions.

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